FILTRATION OF GAS-CONDENSATE MIXTURES IN A POROUS MEDIUM

(O FILTRATSII GAZOKONDENSATNYIKH SMESEI V PORISTOI SREDE)

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Several cases have occurred recently in practice where the substance to be filtered is frequently in the strata in gaseous form in the initial state (before working). Then, when the pressure is reduced, the heavier components of the substance to be filtered fall out in the liquid state, i.e. as a condensate, a certain proportion of which saturates the rock and is irreversibly lost in the stratum. Until the attainment of a certain degree of saturation of the porous medium by liquid (saturation equilibrium) the liquid remains at rest.

In this work we state the problem of filtration and put forward a solution for the simplest cases of gas-condensate mixture in a porous medium. It should be pointed out that some consideration has already been given to the problem in [1].

1. The gas velocity $\mathbf{v}_1$ and liquid velocity $\mathbf{v}_2$ in inertialess motions of gas-liquid mixtures through porous media are connected to the pressure gradient p and the degree of saturation s of a stratum by liquid by the following relations [1]:

$$\mathbf{v}_1 = -\frac{kf_1(s)}{\mu_1}\operatorname{grad} p, \qquad \mathbf{v}_2 = -\frac{kf_2(s)}{\mu_2}\operatorname{grad} p \qquad (1.1)$$

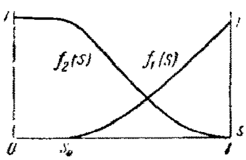

Fig. 1.

In these relations k is the permeability of the porous medium; $f_1(s)$, $f_2(s)$ are the so-called relative phase permeabilities for liquid and gas; μ_1, μ_2 are viscosities of liquid and gas, respectively.

To a first approximation, quantities f_1, f_2, μ_1, μ_2 are considered independent of pressure.

Figure 1 shows typical curves (see, for instance, [2]) of relative phase permeabilities versus degree of saturation. Typical features of these curves are: at saturations less than equilibrium saturation s_0, function $f_1(s)$ is identically equal to zero, i.e. the liquid is stationary. The value of s can vary between 0.2 and 0.5 depending on the rock. Furthermore, when s is small the values of function $f_2(s)$ are close to unity, i.e. the permeability for gas only slightly decreases with fall-out of condensate. Thus, for small saturations we can assume the liquid to be stationary, and the relative gas permeability is taken as unity.

The relation between specific volume V of stabilized condensate, corrected for normal conditions, and the pressure at constant temperature can be represented, approximately, as follows:

$$V = A(p_1 - p), \qquad A = \frac{V_0}{p_1 - p_2} \qquad (p_1 < p < p_2) \tag{1.2}$$

Here V_0 is the maximum specific volume of the condensate, p_1 is the pressure when condensation starts, p_2 is the pressure when condensation is a maximum.

We assume that the gas density does not vary during condensation, and we neglect compressibility of the condensate, both of these assumptions being justified by experiment.

With low saturations it is also possible to neglect variations in rock porosity to the gas and change in the mass of the gas resulting from condensation.

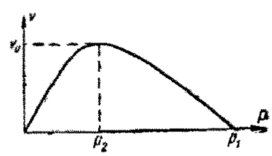

Fig. 2.

Thus, assuming low saturations and the usual characteristics of ideal gas and also isothermal gas flow during filtration, using the equation of gas filtration [3], we can write

$$\frac{\partial p}{\partial t} = a^2 \Delta p^2 \qquad \left(a^2 = \frac{k}{2m\mu_2}\right) \tag{1.3}$$

Let us construct a differential equation to determine the change in saturation. In accordance with (1.2), the volume of condensate falling out in an arbitrary volume of stratum τ in time dt is determined by the volume integral

$$dt \iiint_\tau A \frac{\rho}{\rho_0}\left(\mathrm{v}_2 \cdot \operatorname{grad} p + m \frac{\partial p}{\partial t}\right) d\tau$$

where ρ, ρ_0 are the densities of the gas under strata and normal conditions, respectively. Fall-out of condensate leads to an alteration in the amount of liquid over the interval dt in volume τ by

$$dt \iiint_{\tau} m \frac{\partial s}{\partial t} d\tau$$

If we equate these quantities and make use of arbitrary volume τ, we arrive at the required equation for the degree of saturation in the form

$$\frac{\partial s}{\partial t} = A \frac{\rho}{\rho_0} \left(\frac{\mathbf{v}_2}{m} \cdot \operatorname{grad} p + \frac{\partial p}{\partial t} \right) \tag{1.4}$$

In accordance with the above, let us assume that at the initial instant the saturation is zero everywhere in the stratum; then on integrating Equation (1.4) we find

$$s = A \int_0^t \frac{\rho}{\rho_0} \left(\frac{\mathbf{v}_2}{m} \cdot \operatorname{grad} p + \frac{\partial p}{\partial t} \right) dt \tag{1.5}$$

In view of (1.1) and because of low saturation, this equation can be reduced to this form:

$$s = \frac{Ak}{m\mu_2} \int_0^t \frac{\rho}{\rho_0} (\operatorname{grad} p)^2 \, dt + A \int_0^t \frac{\rho}{\rho_0} \frac{\partial p}{\partial t} dt \tag{1.6}$$

However, bearing in mind that the gas is ideal and the motion is isothermal

$$\frac{\rho}{\rho_0} = \frac{T_0}{p_0 T_1} p$$

where p_0, T_0 are pressure and temperature corresponding to normal conditions, and T_1 is the stratum temperature, so that Equation (1.6) reduces to the final form

$$s = b \int_0^t p (\operatorname{grad} p)^2 \, dt + c \left[p^2 (x, y, z, t) - p^2 (x, y, z, 0) \right] \tag{1.7}$$

$$\left(b = \frac{AkT_0}{\mu_2 m p_0 T_1}, \quad c = \frac{AT_0}{2p_0 T_1} \right)$$

Thus, if we know the solution to the problem of gas motion with the given boundary and initial conditions, we can find the saturation distribution for the motion of gas-condensate mixture from Formula (1.7).

2. We will now deal with plane radial filtration of gas-condensate mixture to a single pore of small radius in an infinite stratum.

Pressure $p(r, t)$ and saturation $s(r, t)$ satisfy the equations

$$\frac{\partial p}{\partial t} = a^2 \left(\frac{\partial^2 p^2}{\partial r^2} + \frac{1}{r} \frac{\partial p^2}{\partial r} \right), \qquad \frac{\partial s}{\partial t} = bp \left(\frac{\partial p}{\partial r} \right)^2 + c \frac{\partial p^2}{\partial t} \tag{2.1}$$

We specify the following boundary and initial conditions:

$$p(r, 0) = p_1, \ p(\infty, t) = p_1, \ s(r, 0) = 0 \tag{2.2}$$

$$\lim_{r \to r_0} 2\pi r h \frac{pT_0}{p_0 T_1} \frac{k}{\mu_2} \frac{\partial p(r, t)}{\partial r} = q_0 = \text{const} \tag{2.3}$$

where h is the thickness of the stratum, r_0 is the radius of the pore and q_0 is the gas flow or discharge reduced to normal conditions.

Both here and in the following discussions the problem will be simplified by dealing with the case where initial stratum pressure is equal to the pressure when condensation starts.

Using the Π-theorem [5], the solutions of system (2.1) are represented as follows:

$$\frac{p(r, t)}{p_1} = F_1(\xi, \lambda), \qquad \xi = \frac{r}{a\sqrt{p_1 t}}, \quad \lambda = \frac{q_0 \mu_2 p_0 T_1}{\pi k h p_1^2 T_0} \tag{2.4}$$

$$s(r, t) = F_2(\xi, \lambda, \delta_1, \delta_2); \quad \delta_1 = \frac{b p_1^2}{a^2}, \qquad \delta_2 = 2 c p_1^2 \tag{2.5}$$

It was demonstrated in [4] that the first equation of (2.1) can be linearized with sufficient accuracy as suggested by Leibenzon [3]. Then for F_1 we have

$$F_1 = \sqrt{1 - {}^1/_2 \lambda \operatorname{Ei}(-{}^1/_8 \xi^2)} \tag{2.6}$$

If we insert (2.4) and (2.5) into the second of Equations (2.1) we arrive at the following ordinary differential equation for F_2:

$$2\delta_1 F_1 \left(\frac{dF_1}{d\xi}\right)^2 + \xi \frac{dF_2}{d\xi} - \delta_2 F_1 \xi \frac{dF_1}{d\xi} = 0 \tag{2.7}$$

Inserting (2.6) into (2.7) and taking account of conditions $F_1(\infty, \lambda) = 1$ and $F_2(\infty, \lambda, \delta_1, \delta_2) = 0$, we obtain

$$F_2 = \int_\xi^\infty \frac{\delta_1 \lambda^2}{2} \frac{1}{\xi^3} \frac{\exp(-{}^1/_4 \xi^2)\, d\xi}{\sqrt{1 - {}^1/_2 \lambda \operatorname{Ei}(-{}^1/_8 \xi^2)}} - \frac{\delta_2 \lambda}{4} \operatorname{Ei}\left(-\frac{1}{8}\xi^2\right) \tag{2.8}$$

3. We will discuss the transient filtration of a gas-condensate mixture into a straight gallery located in a semi-infinite stratum.

Pressure $p(x, t)$ and saturation $s(x, t)$ satisfy the system

$$\frac{\partial p}{\partial t} = a^2 \frac{\partial^2 p^2}{\partial x^2}, \qquad \frac{\partial s}{\partial t} = bp\left(\frac{\partial p}{\partial x}\right)^2 + c\frac{\partial p^2}{\partial t} \tag{3.1}$$

We look for the solution to this system with the conditions

$$p(0,t)=p_3, \qquad p(\infty,t)=p_1, \qquad p(x,0)=p_1, \qquad s(x,0)=0 \qquad (3.2)$$

Because of the Π-theorem the solution of system (3.1) embodies similarity and has the form

$$\frac{p(x,t)}{p_1}=F_1\left(\xi,\ \frac{p_3}{p_1}\right) \quad \left(\xi=\frac{x}{a\sqrt{p_1 t}}\right), \qquad s(x,t)=F_2\left(\xi,\ \frac{p_3}{p_1}\ \delta_1,\delta_2\right) \quad (3.3)$$

If we insert (3.3) into (3.1) we obtain

$$\frac{d^2F_1{}^2}{d\xi^2}+\frac{1}{2}\,\xi\,\frac{dF_1}{d\xi}=0, \qquad 2\delta_1F_1\left(\frac{dF_1}{d\xi}\right)^2+\xi\,\frac{dF_2}{d\xi}-\delta_2F_1\xi\,\frac{dF_1}{d\xi}=0 \quad (3.4)$$

Conditions (3.2) take the form

$$F_1\left(\infty,\frac{p_3}{p_1}\right)=1, \quad F_1\left(0,\ \frac{p_3}{p_1}\right)=\frac{p_3}{p_1}, \quad F_2\left(\infty,\frac{p_3}{p_1},\ \delta_1,\delta_2\right)=0 \quad (3.5)$$

Results of a numerical solution to the first of Equations (3.4) with the given conditions are presented in [6].

From the second equation (3.4) we obtain by integration

$$F_2=2\delta_1\int_{\xi}^{\infty}\frac{F_1}{\xi}\left(\frac{dF_1}{d\xi}\right)^2 d\xi+\frac{\delta_2}{2}\,[F_1{}^2(\xi)-1)] \qquad (3.6)$$

4. For solving the problem of Section 2, let us use the method of progressive change in steady-state conditions. We deal with the so-called first phase.

The pressure distribution with steady gas filtration is determined by the formula (see, for instance, [2])

$$p(r,t)=p_1\sqrt{1-\lambda\ln\frac{R}{r}} \qquad (r_0\leqslant r\leqslant R(t)) \qquad (4.1)$$

If, in the second equation of (2.1), we substitute instead of $p(r, t)$ its expression (4.1) we arrive at

$$\frac{\partial s}{\partial t}=\frac{b\lambda^2p_1{}^3}{4r^2\sqrt{1-\delta_1\ln(R/r)}} \qquad (4.2)$$

For the first phase the relation $R(t)$ is found from the expression (see, for instance, [7])

$$t=\frac{m\mu_2r_0{}^2}{4kp_1}\left[\left(\frac{R}{r_0}\right)^2-2\ln\left(\frac{R}{r_0}\right)-1\right] \qquad (4.3)$$

If we integrate Equation (4.2) and neglect small second-order quantities we obtain

$$s = \frac{q_0\mu_2 A}{4\pi kh}\left(\frac{R}{r}\right)^2\left\{1+\frac{1}{4}\lambda-\frac{1}{2}\lambda\ln\frac{R}{r}-\sqrt{1-\lambda\ln\frac{R}{r}}\right\} \quad (4.4)$$

In Fig. 3 results are shown of numerical calculations using Formula (4.4) for the case $q_0 = 10^5$ m^3 per 24 hr. day, $m = 0.2$, $h = 20$ m, $p_1 = 360$ atm, $P_2 = 60$ atm abs, $r_0 = 0.1$ m, $k = 20$ md, $\mu_2 = 0.028$ cn, $V_0 = 120$ cm^3/Nm^3, $T_0 = T_1$ and for various values of R (in meters) indicated on the curves.

For clarity, the values of r are shown on the graph.

5. We now deal with the more general case of gas-condensate filtration when the liquid phase is also in motion.

By analogy with gaseous liquid filtration, in this case the following differential equations can be derived:

$$m\frac{\partial s}{\partial t} = -\operatorname{div}\mathbf{v}_1 + A\frac{\rho}{\rho_0}\left[m(1-s)\frac{\partial p}{\partial t}+\mathbf{v}_2\cdot\operatorname{grad}p\right] \quad (5.1)$$

$$\operatorname{div}(\rho\mathbf{v}_2) = m\frac{\partial}{\partial t}[\rho(1-s)] \quad (5.2)$$

An assumption has been made in deriving (5.2), namely, that the mass of condensate in the gas is small as compared with the mass of the gas.

In the case of steady or stable filtration Equations (5.1) and (5.2) take the form

$$-\operatorname{div}\mathbf{v}_1 + A\frac{\rho}{\rho_0}\mathbf{v}_2\cdot\operatorname{grad}p = 0; \qquad \operatorname{div}\rho\mathbf{v}_2 = 0 \quad (5.3)$$

Following Khristianovich [8] let us observe the change in the gas-condensate factor, which is the ratio of gas flow under normal conditions Q_2 to the flow or discharge of condensate Q_1 along a stream line.

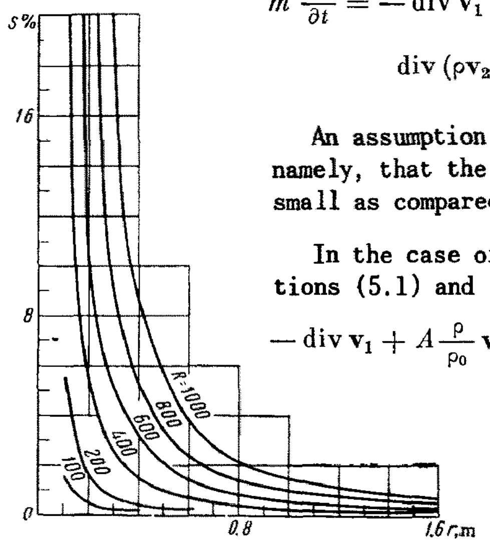

Fig. 3.

It is clearly evident that the first equation of (5.3) can be put in the following way:

$$\rho\mathbf{v}_2\cdot\operatorname{grad}\left[-\frac{f_1(s)\mu_2}{f_2(s)\mu_1\rho}+A\frac{p}{\rho_0}\right]+\left[-\frac{f_1(s)\mu_2}{f_2(s)\mu_1\rho}+A\frac{p}{\rho_0}\right]\operatorname{div}(\rho\mathbf{v}_2)=0 \quad (5.4)$$

Taking into account the second equation of (5.3) and substituting the components of velocity $\mathbf{v}_2$ on the x-, y-, z-axes along a streamline by the quantities dx, dy, dz which are proportional to them, we arrive at

$$d\left[-\frac{f_1(s)\,\mu_2}{f_2(s)\,\mu_1\rho}+A\frac{p}{\rho_0}\right]=0 \tag{5.5}$$

Thus along a streamline we have

$$-\frac{f_1(s)\,\mu_2}{f_2(s)\,\mu_1\rho}+A\frac{p}{\rho_0}=\alpha=\text{const} \tag{5.6}$$

The expression for the gas-condensate factor has the form

$$\Gamma=\frac{Q_2}{Q_1}=\left[\frac{f_1(s)\,\mu_2\rho_0}{f_2(s)\,\mu_1\rho}+Ap_1-Ap\right]^{-1}=[Ap_1-\alpha\rho_0]^{-1}\equiv\text{const} \tag{5.7}$$

With steady filtration, therefore, the gas-condensation factor is constant along a streamline.

From (5.7) we obtain the connection between pressure and degree of saturation.

Following [8], we introduce a new function

$$H=\int_0^p k_2\rho\,dp \tag{5.8}$$

The second equation of (5.3) can then be represented by

$$\Delta H=0 \tag{5.9}$$

Thus the problem of steady filtration of a gas-condensate mixture reduces to that of filtering a homogeneous incompressible liquid. Let us discuss the transient inflow of gas-condensate mixture into a straight gallery located in a semi-infinite stratum.

Pressure $p(x,\ t)$ and saturation $s(x,\ t)$ satisfy the following system of differential equations:

$$-\frac{\partial}{\partial x}\left[f_1(s)\frac{\partial p}{\partial x}\right]+A\beta p\mu_1\frac{m}{k}\left[(1-s)\frac{\partial p}{\partial t}+\frac{k}{m\mu_2}f_2(s)\left(\frac{\partial p}{\partial x}\right)^2\right]=\frac{m\mu_1}{k}\frac{\partial s}{\partial t}$$

$$\frac{\partial}{\partial x}\left[pf_2(s)\frac{\partial p}{\partial x}\right]=\frac{m\mu_2}{k}\frac{\partial}{\partial t}[p(1-s)]\qquad\left(\beta=\frac{T_0}{p_0T_1}\right) \tag{5.10}$$

We specify the following conditions

$$p(0,t)=p_3,\qquad p(x,0)=p_1,\qquad p(\infty,t)=p_1,\quad s(x,0)=0 \tag{5.11}$$

Because of the Π-theorem the systems (5.1) embody similarity and have the form

$$\frac{p}{p_1} = F_1\left(\xi, \alpha_1, \alpha_2, \frac{p_3}{p_1}\right) \qquad \left(\xi = \frac{x}{a\sqrt{p_1 t}}\right)$$
$$S = F_2\left(\xi, \alpha_1, \alpha_2, \frac{p_3}{p_1}\right) \qquad \left(\alpha_1 = A\beta p_1^2,\ \alpha_2 = \frac{\mu_1}{\mu_2}\right) \tag{5.12}$$

If we insert (5.12) into (5.10) we obtain the following system of ordinary differential equations for F_1 and for F_2:

$$f_1'(S)\frac{dF_2}{d\xi}\frac{dF_1}{d\xi} + f_1(S)\frac{d^2F_1}{d\xi^2} - \frac{\alpha_1\alpha_2}{4}F_1\left[4f_2(S)\left(\frac{dF_1}{d\xi}\right)^2 - (1-S)\,\xi\frac{dF_1}{d\xi}\right] = \frac{\alpha_2}{4}\,\xi\,\frac{dF_2}{d\xi} \tag{5.13}$$

$$f_2(S)F_1\frac{d^2F_1}{d\xi^2} + f_2'(S)F_1\frac{dF_2}{d\xi}\frac{dF_1}{d\xi} + f_2(S)\left(\frac{dF_1}{d\xi}\right)^2 = -\frac{1}{4}\,\xi\,\frac{d\left[F_1(1-F_2)\right]}{d\xi}$$

Conditions (5.11) take the following form:

$$F_1\left(0, \alpha_1, \alpha_2, \frac{p_3}{p_1}\right) = \frac{p_3}{p_1}, \quad F_1\left(\infty, \alpha_1, \alpha_2, \frac{p_3}{p_1}\right) = 1, \quad F_2\left(\infty, \alpha_1, \alpha_2, \frac{p_3}{p_1}\right) = 0 \tag{5.14}$$

In order to be able to solve system (5.13) with conditions (5.14) it is necessary to resort to one of the methods of numerical integration.

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BIBLIOGRAPHY

1. Masket, M., *Fizicheskie osnovy tekhnologii dobychi nefti (Fundamentals of the Physics of Petroleum Extraction Technology)*. Gostoptekhizdat, 1953.

2. Shchelkachev, V.N. and Lapuk, B.B., *Podzemnaia gidravlika (Subterranean Hydraulics)*. Gostoptekhizdat, 1949.

3. Leibenzon, L.S., *Dvizhenie prirodnykh zhidkostei i gazov v poristoi srede (Motion of Natural Liquids and Gases through a Porous Medium)*. GITTL, 1947.

4. Barenblatt, G.I., O vozmozhnosti linearizatsii v nekotorykh zadachakh nestatsionarnoi filtratsii gaza (Possibility of linearization in several transient gas-filtration problems). *Izv. Akad. Nauk SSSR, OTN* No. 11, 1956.

5. Sedov, L.I., *Metody podobia i razmernosti v mekhanike (Similarity and Dimensional Methods in Mechanics)*. GITTL, 1951.

6. Polubarinova-Kochina, P.Ia., *Teoriia dvizheniia gruntovykh vod (Theory of Motion of Ground Water)*. GITTL, 1952.

7. Charnyi, I.A., *Osnovy podzemnoi gidravliki (Fundamentals of Subterranean Hydraulics)*. Gostoptekhizdat, 1956.

8. Khristianovich, S.A., O dvizhenii gazirovannoi zhidkosti v poristykh porodakh (Motion of gaseous liquids through porous rocks). *PMM* Vol. 5, No. 2, 1941.

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